

Name:

Date:

Period: 6

Surface Area and Volume of Spheres

on Objective INBAT find surface areas + volumes of spheres

Sphere: the set of all points in space equidistant from a given point, the center of the sphere.

A radius of a sphere is a segment from the center to a point on the sphere.

A chord of a sphere is a segment whose endpoints are on the sphere.

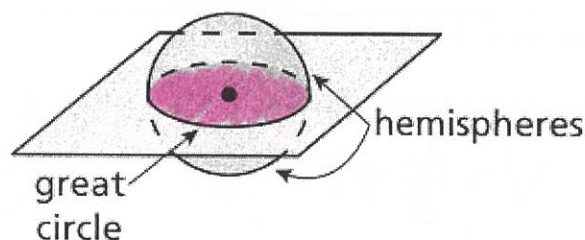
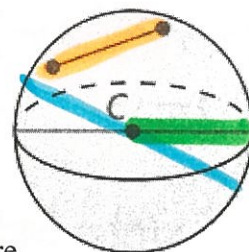
A diameter of a sphere is a chord that contains the center.

→ As with circles, the diameter of a sphere is twice the length of the radius of the sphere.

Great Circle: If a plane intersects a sphere and contains its center, the intersection is called a great circle of the sphere.

→ The circumference of the great circle is the circumference of the sphere

→ Every great circle separates the sphere into two congruent halves, called hemispheres.

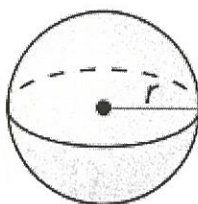


Surface Area of a Sphere

surface area S of a sphere is

$$S = 4\pi r^2$$

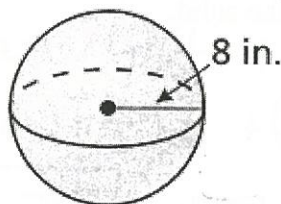
where r is the radius of the sphere



**The proof is relatively difficult...if you would like to review an explanation of how this formula came to be, please refer to the link on the right side of the Edline home page ☺

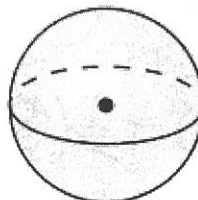
Examples:

1. Find the surface area of the sphere.



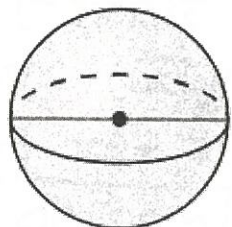
$$\begin{aligned} S &= 4\pi(8)^2 \\ &= 256\pi \text{ in}^2 \\ &\approx 804.25 \text{ in}^2 \end{aligned}$$

2. Find the surface area of the sphere.



$$\begin{aligned} C &= 12\pi \text{ ft} = 2\pi r \\ r &= 6 \\ S &= 4\pi(6)^2 \\ &= 144\pi \text{ ft}^2 \\ &\approx 452.39 \text{ ft}^2 \end{aligned}$$

3. Find the diameter of the sphere.



$$S = 20.25\pi \text{ cm}^2$$

$$\begin{aligned} \frac{4\pi r^2}{4\pi} &= \frac{20.25\pi}{4\pi} \\ \sqrt{r^2} &= \sqrt{5.0625} \end{aligned}$$

$$r = 2.25 \rightarrow d = 4.5 \text{ cm}$$



$$\begin{aligned} &2\pi r^2 + \pi r^2 \\ &\quad \uparrow \quad \quad \uparrow \\ &\text{curvy} \quad \text{great} \end{aligned}$$

$$\begin{aligned} &2(\pi(2)^2) + \pi(2)^2 \\ &8\pi + 4\pi = 12\pi \end{aligned}$$

Watch the YouTube video (link posted on right side of Edline page)

The cylinder, sphere, and cone all have the same diameter and same height.

What combination of sphere volume and cone volume did it take to fill the cylinder?

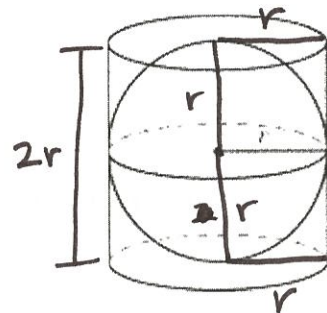
1 spheres, 1 cones

So... $\text{vol. cyl} = \text{vol. sphere} + \text{vol. cone}$

$$(\pi r^2)(2r) = \text{vol. sphere} + \frac{1}{3}(\pi r^2)(2r)$$

$$2\pi r^3 = \text{vol. sphere} + \frac{2\pi r^3}{3}$$

$$\text{vol. sphere} = 2\pi r^3 - \frac{2\pi r^3}{3} = \frac{6\pi r^3}{3} - \frac{2\pi r^3}{3} = \boxed{\frac{4\pi r^3}{3}}$$

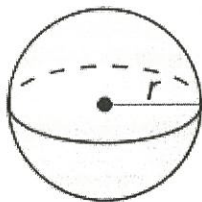


Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere



Examples:

5. The surface area of a sphere is 324π sq. cm. Find the volume of the sphere.

$$4\pi r^2 = 324\pi$$

$$r^2 = 81$$

$$r = 9$$

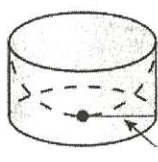
$$V = \frac{4}{3}\pi(9)^3$$

$$V = \frac{4}{3}\pi(729)$$

$$V = \frac{2916\pi}{3}$$

$$\boxed{V = 972\pi \text{ cm}^3}$$

6. Find the volume of the solid.



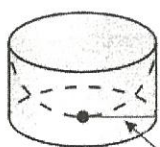
$$\text{cyl} - \frac{1}{2}\text{sphere}$$

$$\pi(2)^2(2) - \frac{1}{2}\left(\frac{4}{3}\pi(2)^3\right)$$

$$8\pi - \frac{16\pi}{3}$$

$$\frac{24\pi}{3} - \frac{16\pi}{3} = \boxed{\frac{8\pi}{3} \text{ in}^3}$$

7. Find the surface area of the solid.



$$\text{cyl} + \frac{1}{2}\text{sphere}$$

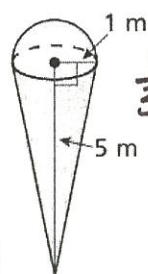
$$B + Ph + \frac{1}{2}(4\pi r^2)$$

$$\pi(2)^2 + 2(\pi(2))(2) + 2\pi(2)^2$$

$$4\pi + 8\pi + 8\pi$$

$$\boxed{= 20\pi \text{ in}^2}$$

8. Find the volume of the solid.

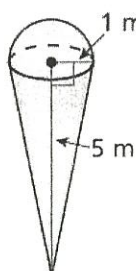


$$\text{cone} + \frac{1}{2}\text{sphere}$$

$$\frac{1}{3}\pi(1)^2(5) + \frac{1}{2}\left(\frac{4}{3}\pi(1)^3\right)$$

$$\frac{5\pi}{3} + \frac{2\pi}{3} = \boxed{\frac{7\pi}{3} \text{ m}^3}$$

9. Find the surface area of the solid.



$$\frac{1}{2}Pl + \frac{1}{2}(4\pi r^2)$$

$$\frac{1}{2}(2\pi(1)(\sqrt{26}) + 2\pi(1)^2)$$

$$= \sqrt{26}\pi + 2\pi \text{ m}^2$$

$$\boxed{\approx 22.30 \text{ m}^2}$$

$$* l = \sqrt{26}$$